

THE MODIFICATION OF THE NONLINEAR GUIDING CENTER THEORY

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ABSTRACT

We modify the NonLinear Guiding Center (NLGC) theory (Matthaeus et al. 2003) for perpendicular diffusion by replacing the spectral amplitude of the two-component model magnetic turbulence with the 2D component one (following Shalchi 2006), and replacing the constant a^2 , indicating the degree particles following magnetic field line, with a variable a'^2 as a function of the magnetic turbulence. We combine the modified model with the NonLinear PArallel (NLPA) diffusion theory (Qin 2007) to solve perpendicular and parallel diffusion coefficients simultaneously. It is shown that the new model agrees better with simulations. Furthermore, we fit the numerical results of the new model with polynomials, so that parallel and perpendicular diffusion coefficients can be calculated directly without iteration of integrations, and many numerical calculations can be reduced.

Subject headings:

1. INTRODUCTION

Knowledge of charged energetic particles' diffusion mechanism is necessary to study the transport and acceleration of cosmic rays. Matthaeus et al. (2003) developed a NonLinear Guiding Center (NLGC)¹ theory to describe the perpendicular diffusion coefficient, which is written as (Matthaeus et al. 2003)

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2} \int d^3 \mathbf{k} \frac{S_{xx}(\mathbf{k})}{\frac{v}{\lambda_{\parallel}} + k_{\perp}^2 \kappa_{xx} + k_{\parallel}^2 \kappa_{zz} + \gamma(\mathbf{k})}, \quad (1)$$

where the parameter $a^2 = 1/3$ indicating the degree particles following Magnetic Field Line (MFL). In addition, the spectral amplitudes of two-component model turbulence $S_{xx}(\mathbf{k})$ is the sum of that of 2D component $S_{xx}^{2D}(\mathbf{k})$ and slab component $S_{xx}^{slab}(\mathbf{k})$ (e.g., Bieber et al. 1996),

$$\begin{aligned} S_{xx}(\mathbf{k}) &= S_{xx}^{2D}(\mathbf{k}) + S_{xx}^{slab}(\mathbf{k}) \\ &= S_{xx}'^{2D}(k_{\perp}) \frac{2k_y^2 \delta(k_{\parallel})}{\pi k_{\perp}^3} + S_{xx}'^{slab}(k_{\parallel}) \frac{\delta(k_{\perp})}{2\pi k_{\perp}}, \end{aligned} \quad (2)$$

with 2D component $S_{xx}'^{2D}(k_{\perp})$ and slab component $S_{xx}'^{slab}(k_{\parallel})$ written as

$$S_{xx}'^{2D}(k_{\perp}) = C(\nu) \lambda_{2D} \langle b_{2D}^2 \rangle (1 + k_{\perp}^2 \lambda_{2D}^2)^{-\nu} \quad (3)$$

$$S_{xx}'^{slab}(k_{\parallel}) = C(\nu) \lambda_{slab} \langle b_{slab}^2 \rangle (1 + k_{\parallel}^2 \lambda_{slab}^2)^{-\nu}, \quad (4)$$

and $C(\nu) = \frac{1}{2\sqrt{\pi}} \frac{\Gamma(\nu)}{\Gamma(\nu-1/2)}$. Here, λ_{slab} and λ_{2D} are the spectral bend-over scales of slab and 2D components of turbulence, respectively, and particles parallel mean free path is related to the parallel diffusion coefficient as $\lambda_{\parallel} = 3\kappa_{zz}/v$. It is considered that NLGC is the first perpendicular diffusion theory that agrees well with simulations and spacecraft observations in typical solar wind conditions (Zank et al. 2004; Bieber et al. 2004). Note that in Eqs. (3) and (4) the spectra of 2D and slab components, respectively, are all set flat in energy range

¹Note that the acronyms in this paper are briefly explained in Table 1.

with lower wavenumbers. However, Matthaeus et al. (2007) suggested that in energy range, $k_{\perp} \ll 1/\lambda_{2D}$, the spectrum of 2D component should not be constant. In addition with a more general form of the 2D spectrum with energy range spectrum index q , Shalchi et al. (2010) found that the behavior of the spectrum in energy range is important to determine the perpendicular diffusion.

Furthermore, the NLGC model is an integral equation for perpendicular diffusion coefficient with parallel diffusion coefficient as an input, so it is difficult to include NLGC in a numerical model to study energetic particles transport or acceleration. Zank et al. (2004) and Shalchi et al. (2004a) derived explicit expressions for the perpendicular diffusion coefficient, where parallel diffusion coefficient from Quasilinear Theory (QLT Jokipii 1966) can be used as input. Furthermore, Shalchi et al. (2004b) developed a nonlinear model, WNLT, to describe parallel and perpendicular diffusion simultaneously. But the WNLT model is complicated and difficult to use in numerical models.

Moreover, because of nonlinear effects, QLT is not very accurate compared to the simulation results (e.g., Qin et al. 2002). Based on the NLGC theory, a nonlinear model, the NLPA theory of parallel diffusion coefficient is derived as (Qin 2007)

$$\kappa_{zz} = \left\{ 6a_x \left(\frac{\Omega}{v} \right)^2 \int d^3\mathbf{k} \frac{S_{xx}(\mathbf{k})}{B_0^2} \times \frac{\frac{v}{\lambda_{\parallel}} + k_{\perp}^2 \kappa_{xx} + k_{\parallel}^2 \kappa_{zz}}{\Omega^2 + [\frac{v}{\lambda_{\parallel}} + k_{\perp}^2 \kappa_{xx} + k_{\parallel}^2 \kappa_{zz} + \gamma(\mathbf{k})]^2} \right\}^{-1}, \quad (5)$$

with the parameter

$$a_x = \frac{1}{2} \sqrt{\frac{\tilde{E}_s}{[\xi/(1+\xi)](1/\tilde{b}) + \tilde{b}/(2\xi)}}, \quad (6)$$

and $\tilde{r} = 2\pi r_L/\lambda_c$, $\tilde{b} = b/B_0$, $\xi = \tilde{r}/\tilde{b}$, $\tilde{E}_s = E_{slab}/E_{total}$, and $E_{total} = E_{slab} + E_{2D}$. Here, the correlation length of the slab turbulence λ_c is related to the slab turbulence correlation scale λ_{slab} as $\lambda_c = 2\pi C(\nu)\lambda_{slab}$, r_L is the particle maximum gyro-radius, $b = (\langle b_{slab}^2 \rangle + \langle b_{2D}^2 \rangle)^{1/2}$, b/B_0 is the turbulence level, and $E_{slab} = \langle b_{slab}^2 \rangle$ and $E_{2D} = \langle b_{2D}^2 \rangle$ are magnetic turbulence

energy in slab and 2D components, respectively. Note that in equation (8) of Qin (2007) there was a typo in the definition of the parameter a_x which was corrected in Qin (2013). It is also noted that QLT (Jokipii 1966) can not be obtained from this theory in the corresponding limit. Again, NLPA model is an integral equation for parallel diffusion coefficient with perpendicular diffusion coefficient as an input. Furthermore, the NLGC and NLPA theories can be combined to get the NLGC-E model to determine the parallel and perpendicular diffusion coefficients simultaneously (Qin 2007).

In addition, Shalchi (2010) developed a unified diffusion theory for perpendicular diffusion based on Matthaeus et al. (NLGC, 2003), with the Fokker-Planck equation to compute the fourth-order correlations during derivation. Furthermore, Tautz and Shalchi (2011) compared the unified diffusion theory, noted as INLGC, (as well as NLGC) with simulations. It is shown that the unified diffusion theory, INLGC, can be used for 3D turbulence. Moreover, INLGC automatically satisfies the subdiffusive result for slab turbulence ($\kappa_{\perp} = 0$), and corresponds to NLGC for two-component turbulence without any additional assumptions.

In this study, for simplicity purpose, we modify the NLGC theory directly by replacing the spectral amplitude of the two-component model magnetic turbulence with the 2D component one and replacing the constant indicating the degree particles following magnetic field line with a variable of magnetic turbulence. We also fit the numerical results of the modified model with polynomials. The paper is organized as follows. We discuss the modification of the NLGC theory in section 2. The polynomial fitting of the new model is discussed in section 3. Finally, conclusions are presented in section 4.

2. MODIFICATION OF THE NLGC THEORY

The NLGC theory agrees with simulation results very well in general solar wind conditions. However, from simulation results in Qin (2007), especially in Figure 3 of Qin (2007), we find that the NLGC theory for perpendicular diffusion does not agree simulation results well when the turbulence is nearly pure slab or pure 2D, so it is necessary for one to modify the NLGC theory. Firstly, it is considered that the slab component of the turbulent magnetic field does not contribute directly to the perpendicular diffusion (Shalchi 2006, 2010). Therefore, in the NLGC theory of the perpendicular diffusion coefficient equation (1), the spectral amplitude of the two-component model magnetic turbulence $S_{xx}(\mathbf{k})$ should only include the 2D component, $S_{xx}^{2D}(\mathbf{k})$ (Shalchi 2006). Secondly, we assume the degree particles following MFL is varied with the conditions of magnetic turbulence, so we modify the parameter a^2 in equation (1) with different forms and compare with simulation results in Qin (2007), and the best form we can get so far is

$$a'^2 = \left(\sqrt{\frac{\lambda_{2D}}{\lambda_{slab}}} \frac{E_{total}}{E_{slab}} + \frac{4}{3} \frac{E_{total}}{E_{2D}} \right)^{-1}. \quad (7)$$

Therefore, by replacing $S_{xx}(\mathbf{k})$ and a^2 with $S_{xx}^{2D}(\mathbf{k})$ and a'^2 , respectively, in equation (1), we get a modified NLGC theory for perpendicular diffusion coefficient,

$$\begin{aligned} \kappa_{xx} = & \frac{a'^2 v^2}{3} \int dk_{\perp} 2C(\nu) \lambda_{2D} \frac{\langle b_{2D}^2 \rangle}{B_0^2} (1 + k_{\perp}^2 \lambda_{2D}^2)^{-\nu} \\ & \times \frac{1}{\frac{v}{\lambda_{\parallel}} + k_{\perp}^2 \kappa_{xx} + \gamma(\mathbf{k})}. \end{aligned} \quad (8)$$

Here, symmetric 2D component turbulence is assumed. In addition, the 2D spectrum of Eq. (3) with energy range spectrum index $q = 0$ is used in order to compare with simulations in Qin (2007). In the future, a more general form of 2D spectrum with $q \neq 0$ can be used. In the following, we note this modified model as NLGC-N. In addition, we combine the NLGC-N and NLPA models, the equations (8) and (5), respectively, to get an NLGCE-N model. Next we compare the numerical results of NLGCE-N model with that of NLGC-E

and the simulation results from Qin (2007). Here, $\gamma(\mathbf{k})$ is chosen to be 0 for static magnetic turbulence, and ν is chosen to be $5/6$. In addition, we can also define a parameter, the perpendicular mean free path $\lambda_{\perp} \equiv 3\kappa_{zz}/v$ for simplicity purpose.

Top and bottom panels of Figure 1 show perpendicular and parallel mean free paths, respectively, as a function of E_{slab}/E_{total} , with $r_L/\lambda_c = 0.048$, $b/B_0 = 1$ and $\lambda_{2D}/\lambda_{slab} = 0.1$. Diamonds are from the simulations in Qin (2007), Dotted, dashed, and dashed-dotted lines indicate results from NLGC-E, NLGCE-N, and NLGCE-F, respectively. Later we will study NLGCE-F, which is the polynomial fitting of NLGCE-N. The simulation results in Figure 1 are obtained from the Figure 3 of Qin (2007). As already shown in Qin (2007), the perpendicular diffusion coefficient from NLGC-E agrees well with simulation results when $E_{slab}/E_{total} \sim 0.2$, but it does not agree well with simulations when $E_{slab}/E_{total} \ll 0.2$ or $E_{slab}/E_{total} \rightarrow 1$. However, the perpendicular diffusion coefficient from the new model, NLGCE-N, agrees very well for the whole range of E_{slab}/E_{total} , from $E_{slab}/E_{total} \ll 0.2$ to $E_{slab}/E_{total} \rightarrow 1$. At the same time, the parallel diffusion coefficient from NLGCE-N generally agrees well with simulations. Although with $E_{slab}/E_{total} \ll 0.02$, the NLGCE-N is relatively worse than NLGC-E, the results of NLGCE-N are still acceptable.

Figure 2 is similar to Figure 1, except that x-axis is $\lambda_{2D}/\lambda_{slab}$ with $E_{slab}/E_{total} = 0.2$, and that the simulation results are obtained from the Figure 4 of Qin (2007). From the bottom panel of Figure 2 we can see that both NLGC-E and NLGCE-N agree well with simulations in parallel diffusion. However, from the top panel of Figure 2 the two models are different in perpendicular diffusion. NLGC-E agrees well with simulations in perpendicular diffusion coefficient with $\lambda_{2D}/\lambda_{slab} \sim 0.01$, but the agreement becomes worse as $\lambda_{2D}/\lambda_{slab}$ increases. On the other hand, to compare to simulations, NLGCE-N is worse than NLGC-E in perpendicular diffusion with $\lambda_{2D}/\lambda_{slab} \lesssim 0.02$, but it is better than NLGC-E with $\lambda_{2D}/\lambda_{slab} \gtrsim 0.03$. Generally, NLGCE-N agrees better with simulations than NLGC-E in

perpendicular diffusion.

Figure 3 is similar to Figure 1, except that x-axis is r_L/λ_c with $E_{slab}/E_{total} = 0.2$, and that the simulation results are obtained from the Figure 1 of Qin (2007). From the Figure 3 we can see that both NLGC-E and NLGCE-N agree well with simulations. In addition, Figure 4 is similar to Figure 1, except that x-axis is r_L/λ_c with $E_{slab}/E_{total} = 0.2$ and $b/B_0 = 0.2$, and that the simulation results are obtained from the Figure 2 of Qin (2007). Again, from the Figure 4 we can see that both NLGC-E and NLGCE-N agree well with simulations.

Therefore, we show that the new NLGCE-N model is improved compared to the NLGC-E model, especially when the magnetic turbulence is nearly pure slab or 2D.

In addition, we compare the modified model NLGC-N with the INLGC model (Shalchi 2010; Tautz and Shalchi 2011). Since the NLGC-N and INLGC are models for perpendicular diffusion with parallel diffusion coefficients as input, we use κ_{\parallel} from the simulation results in Qin (2007) as input. Left and right panels of Figure 5 are similar as top panels of Figure 1 and Figure 2, respectively, except that dashed and dashed-dotted lines indicate results from NLGC-N and INLGC, respectively. From left panel of the figure we can see that when $0.1 < E_{slab}/E_{total} \lesssim 0.5$, both NLGC-N and INLGC agree well with simulations. But when $E_{slab}/E_{total} \lesssim 0.1$ or $E_{slab}/E_{total} > 0.5$, NLGC-N agrees better with simulations than INLGC. Furthermore, from right panel of the figure we can see that when $\lambda_{2D}/\lambda_{slab} \sim 0.1$, both NLGC-N and INLGC agree well with simulations, but when $\lambda_{2D}/\lambda_{slab} \not\sim 0.1$, NLGC-N agrees better with simulations than INLGC.

3. POLYNOMIAL FITTING OF NLGCE-N

Although the new NLGCE-N model agrees with simulations well, but numerous iteration of integrations for both of the equations are needed to solve it numerically. In order to simplify the numerical calculations needed using the model to study energetic particles transport or acceleration, we fit the equations of NLGCE-N model with polynomials in parameters of magnetic field and particles, i.e., r_L/λ_{slab} , E_{slab}/E_{total} , b^2/B_0^2 , and $\lambda_{slab}/\lambda_{2D}$, or

$$\ln \frac{\lambda_\alpha}{\lambda_{slab}} = \sum_{i=0}^{n_{\alpha 1}} a_i^\alpha \left(\ln \frac{r_L}{\lambda_{slab}} \right)^i, \quad (9)$$

with

$$\begin{aligned} a_i^\alpha &= \sum_{j=0}^{n_{\alpha 2}} b_{i,j}^\alpha \left(\ln \frac{E_{slab}}{E_{total}} \right)^j \\ b_{i,j}^\alpha &= \sum_{k=0}^{n_{\alpha 3}} c_{i,j,k}^\alpha \left(\ln \frac{b^2}{B_0^2} \right)^k \\ c_{i,j,k}^\alpha &= \sum_{l=0}^{n_{\alpha 4}} d_{i,j,k,l}^\alpha \left(\ln \frac{\lambda_{slab}}{\lambda_{2D}} \right)^l \end{aligned}$$

where α indicates \perp or \parallel . Note that this formula is not valid in pure 2D turbulence. By fitting the numerical results of NLGCE-N with the polynomials in wide ranges of parameters, r_L/λ_{slab} , E_{slab}/E_{total} , b^2/B_0^2 , and $\lambda_{slab}/\lambda_{2D}$ as shown in Table 2, we get $\prod_{i=1}^4 (n_{\alpha i} + 1)$ coefficients $d_{i,j,k,l}^\alpha$ for either parallel or perpendicular diffusion. It is noted that with larger values of $n_{\alpha i}$, we can get a fitting formula with higher accuracy, but with more fitting parameters $d_{i,j,k,l}^\alpha$ needed. So we have to balance between accuracy and the simplicity of the fitting formula. We tried fitting formulae with different set of $n_{\alpha i}$ and compared the results with the NLGCE-N carefully (not shown), and we found that with $n_{\alpha 1} = 5$, $n_{\alpha 2} = 3$, $n_{\alpha 3} = 3$, and $n_{\alpha 4} = 2$ for both parallel and perpendicular diffusion, we can get a fitting formula with good accuracy and acceptable calculation scale. This way, we can directly calculate the parallel and perpendicular diffusion coefficients without iteration

of integrations, and the polynomial fitting results of parallel and perpendicular diffusion coefficients are called NLGCE-F. The $\prod_{i=1}^4 (n_{\alpha i} + 1) = 6 \times 4 \times 4 \times 3 = 288$ coefficients $d_{i,j,k,l}^{\alpha}$ for parallel ($\alpha = \parallel$) and perpendicular ($\alpha = \perp$) diffusion in the NLGCE-F model are shown in Tables 3 and 4, respectively.

As an example to show the NLGCE-F's acceptable accuracy with controllable calculation scale, in Figure 6 we show the comparison between the NLGCE-F and the results of a new fitting formula **with** $n_{\alpha 1} = 6$, $n_{\alpha 2} = 4$, $n_{\alpha 3} = 4$, **and** $n_{\alpha 4} = 3$, which is noted as NLGCE-F2. Figure 6 is Similar as Figure 3, except that solid, dotted, and dashed lines indicate results from NLGCE-N, NLGCE-F, and NLGCE-F2, respectively. From top panel of the figure we can see that for perpendicular diffusion, when $r_L/\lambda_c < 0.03$, NLGCE-F2 agrees better with NLGCE-N than NLGCE-F, but when $r_L/\lambda_c > 0.03$, NLGCE-F agrees better with NLGCE-N than NLGCE-F2. However, from bottom panel of the figure we can see that for parallel diffusion, NLGCE-F and NLGCE-F2 agree very well with each other in the range $0.001 \lesssim r_L/\lambda_c \lesssim 0.3$. With comparisons including other variable ranges (not shown) we found that, generally speaking, relative to NLGCE-F2, NLGCE-F is acceptable in agreement with NLGCE-N. But with NLGCE-F2, the number of coefficients $d_{i,j,k,l}^{\alpha}$ for ether parallel or perpendicular diffusion **is** $7 \times 5 \times 5 \times 4 = 700$. Furthermore, in order to show the agreement between the model NLGCE-N and its polynomial fitting NLGCE-F, in Figures 1-4, we plot the results of NLGCE-F with dashed-dotted lines. From the figures we can see that NLGCE-F agrees with NLGCE-N relatively well.

4. CONCLUSIONS

In this paper, we modified the NLGC model, equation (1), which determines particles perpendicular diffusion, by replacing the spectral amplitude of the two-component model magnetic turbulence $S_{xx}(\mathbf{k})$ with the 2D component one, $S_{xx}^{2D}(\mathbf{k})$ (Shalchi 2006), and

replacing the parameter a^2 with a'^2 which is a function of E_{slab}/E_{total} and $\lambda_{2D}/\lambda_{slab}$, to get a new model NLGC-N for perpendicular diffusion. To combine NLGC-N with NLPA, the model for parallel diffusion, we get a model NLGCE-N, which can be solved simultaneously to describe perpendicular and parallel diffusion. In addition, we show that NLGCE-N agrees better with simulations than NLGC-E, which is the combination of NLGC and NLPA. Furthermore, we fit the numerical results of NLGCE-N with the polynomials in wide ranges of parameters r_L/λ_{slab} , E_{slab}/E_{total} , b^2/B_0^2 , and $\lambda_{slab}/\lambda_{2D}$, to get a new model NLGCE-F. So that we can directly calculate parallel and perpendicular diffusion coefficients simultaneously without iteration of integrations. Therefore, much numerical calculations would be saved to study diffusion coefficients.

It is also noted that when $E_{slab}/E_{total} \lesssim 0.1$ or $\lambda_{2D}/\lambda_{slab} \not\approx 0.1$, the modified model NLGC-N agrees better with simulations than the unified diffusion theory (INLGC). In addition, the modified model NLGC-N is very similar to the two component limit of the unified diffusion theory, with a major difference, i.e., in INLGC a constant parameter $a^2 = 1/3$ is used. So it is suggested that the unified diffusion theory can also adopt the similar modification of parameter a^2 , Eq. (7), as in this paper.

In the future, we would compare our models with simulations with general forms of 2D component with energy range spectrum index $q \neq 0$. In addition, we would use the model NLGCE-F to study transport of energetic particles in solar wind, including solar energetic particles, anomalous cosmic rays, or galactic cosmic rays (e.g., Qin et al. 2006; Zhao et al. 2014). Furthermore, we put the FORTRAN code for NLGCE-F with the data of the coefficients $d_{i,j,k,l}^\alpha$ online at <http://www.qingang.org.cn/code/NLGCE-F> to be freely downloaded and used by anybody.

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Table 1: The notation of terms.

Terms	Explanation	Output	Input	Papers
NLGC	NonLinear Guiding Center theory	κ_{\perp}	κ_{\parallel}	M03 ^a
WNLT	Weakly NonLinear Theory	$\kappa_{\perp}, \kappa_{\parallel}$		S04B ^b
NLPA	NonLinear PArallel diffusion theory	κ_{\parallel}	κ_{\perp}	Q07 ^c
NLGC-E	Combination of NLGC and NLPA	$\kappa_{\perp}, \kappa_{\parallel}$		Q07 ^c
INLGC	Unified diffusion theory	κ_{\perp}	κ_{\parallel}	S10 ^d , TA11 ^e
NLGC-N	Modification of NLGC	κ_{\perp}	κ_{\parallel}	TP ^f
NLGCE-N	Combination of NLGC-N and NLPA	$\kappa_{\perp}, \kappa_{\parallel}$		TP ^f
NLGCE-F	Polynomial fitting of NLGCE-N	$\kappa_{\perp}, \kappa_{\parallel}$		TP ^f

^aMatthaeus et al. (2003)

^bShalchi et al. (2004b)

^cQin (2007)

^dShalchi (2010)

^eTerm noted in Tautz and Shalchi (2011)

^fThis paper

Table 2: The range of the four variences.

$\lambda_{slab}/\lambda_{2D}$	E_{slab}/E_{total}	b^2/B_0^2	r_L/λ_{slab}
$1 \sim 10^3$	$10^{-3} \sim 0.85$	$10^{-4} \sim 10^2$	$10^{-5} \sim 6.3$

Table 3: The coefficients $d_{i,j,k,l}^\alpha$ for parallel diffusion in the NLGCE-F model.

(j k l)	i=0	i=1	i=2	i=3	i=4	i=5
(0 0 0)	0.23553875E+01	0.13029025E+01	0.13427579E+00	-0.20846185E-01	-0.43999458E-02	-0.19732612E-03
(0 0 1)	-0.66243965E-02	0.77768482E-01	0.41205730E-01	0.90071283E-02	0.90300755E-03	0.33110445E-04
(0 0 2)	0.21922888E-02	-0.39684652E-02	-0.17124410E-02	-0.94109440E-04	-0.11972348E-06	-0.45759483E-07
(0 1 0)	-0.14247343E+01	-0.17294922E-01	0.50330910E-01	0.22555091E-02	-0.54744446E-03	-0.36329187E-04
(0 1 1)	0.29375712E-01	-0.10262387E-01	-0.11404086E-01	-0.16183987E-02	-0.55537309E-04	0.92677294E-06
(0 1 2)	-0.10823704E-01	-0.31550348E-02	0.10669635E-02	0.34794916E-03	0.30278040E-04	0.82627464E-06
(0 2 0)	0.96583711E-01	-0.12322827E-01	-0.14806351E-01	-0.19674998E-02	-0.47832394E-04	0.22621783E-05
(0 2 1)	-0.16641267E-01	-0.38108147E-02	0.15277614E-02	0.53316833E-03	0.49867082E-04	0.14719990E-05
(0 2 2)	0.24760055E-02	0.75282162E-03	-0.23891975E-03	-0.11721492E-03	-0.13940055E-04	-0.51898963E-06
(0 3 0)	0.11152796E-01	0.31889965E-03	-0.23482006E-02	-0.47383322E-03	-0.30607870E-04	-0.57224705E-06
(0 3 1)	-0.17297051E-02	-0.54967106E-03	0.19481087E-03	0.72844701E-04	0.72128259E-05	0.22722512E-06
(0 3 2)	0.28333003E-03	0.10946972E-03	-0.23059951E-04	-0.12751296E-04	-0.15280551E-05	-0.56788270E-07
(1 0 0)	-0.14786874E+00	0.35413822E+00	-0.23571521E-01	-0.29883137E-01	-0.41108264E-02	-0.16630590E-03
(1 0 1)	-0.33315220E+00	-0.19457101E+00	0.48492938E-01	0.19439119E-01	0.20217803E-02	0.68237390E-04
(1 0 2)	-0.70191117E-02	0.16863137E-01	-0.42473573E-02	-0.20407538E-02	-0.22445321E-03	-0.77342338E-05
(1 1 0)	-0.12645333E+00	-0.57381385E-01	0.19660793E-01	0.81465626E-02	0.93001667E-03	0.34019143E-04
(1 1 1)	-0.74499928E-01	0.21886029E-01	0.78026833E-02	-0.90778341E-03	-0.27852845E-03	-0.13910525E-04
(1 1 2)	0.88176328E-02	-0.24607326E-02	-0.16399476E-02	-0.65717348E-04	0.18575402E-04	0.12449253E-05
(1 2 0)	0.32714354E-01	-0.96871348E-02	-0.88372387E-02	-0.10864091E-02	0.19497311E-06	0.31551668E-05
(1 2 1)	-0.87371981E-02	0.23907528E-02	-0.84712003E-03	-0.58296444E-03	-0.80491321E-04	-0.32954014E-05
(1 2 2)	0.20518127E-02	0.25128221E-03	-0.21217125E-04	0.70656292E-07	0.91585353E-06	0.56694614E-07
(1 3 0)	0.38644947E-02	0.50051530E-03	-0.11014859E-02	-0.31141696E-03	-0.28640974E-04	-0.86437844E-06
(1 3 1)	0.99272926E-04	0.56214724E-04	-0.18481718E-03	-0.55454476E-04	-0.54975212E-05	-0.18100547E-06
(1 3 2)	0.87113797E-04	0.41654101E-04	0.12554768E-04	0.57474926E-06	-0.11142497E-06	-0.82310051E-08
(2 0 0)	0.12726928E+00	0.60497042E-01	-0.11952481E-01	-0.71946142E-02	-0.90296336E-03	-0.35119674E-04
(2 0 1)	-0.12984315E+00	-0.42695185E-01	0.14716482E-01	0.47345284E-02	0.44192205E-03	0.13655324E-04
(2 0 2)	0.76251665E-02	0.42709723E-02	-0.15275348E-02	-0.53455363E-03	-0.50940514E-04	-0.15623067E-05
(2 1 0)	-0.31863123E-01	-0.19815411E-01	0.52020996E-02	0.28355253E-02	0.35739999E-03	0.13896490E-04
(2 1 1)	-0.16244229E-01	0.93012640E-02	0.16637012E-02	-0.68382140E-03	-0.13628576E-03	-0.62450050E-05
(2 1 2)	0.27835739E-02	-0.97187943E-03	-0.47272017E-03	0.79149520E-05	0.95241782E-05	0.53642837E-06
(2 2 0)	0.46502480E-02	-0.32396413E-02	-0.15041494E-02	-0.75760531E-04	0.16825701E-04	0.12521577E-05
(2 2 1)	0.21825930E-03	0.12337052E-02	-0.50023564E-03	-0.24279710E-03	-0.29501141E-04	-0.11178305E-05
(2 2 2)	0.33644525E-03	0.26880995E-04	0.20519435E-04	0.76226242E-05	0.90872206E-06	0.33814742E-07
(2 3 0)	0.58192239E-03	0.61367478E-04	-0.17658762E-03	-0.52589587E-04	-0.51889990E-05	-0.16995503E-06
(2 3 1)	0.26078224E-03	0.54482841E-04	-0.84274690E-04	-0.24125560E-04	-0.22392916E-05	-0.68572929E-07
(2 3 2)	-0.31587447E-05	0.88101241E-05	0.77940884E-05	0.12632533E-05	0.62626899E-07	0.32419583E-09
(3 0 0)	0.94294495E-02	0.37121738E-02	-0.96294274E-03	-0.49755041E-03	-0.60494434E-04	-0.23180297E-05
(3 0 1)	-0.10376127E-01	-0.26461773E-02	0.11205302E-02	0.31769572E-03	0.27403681E-04	0.78509192E-06
(3 0 2)	0.86310176E-03	0.27169634E-03	-0.12254550E-03	-0.35939188E-04	-0.30618079E-05	-0.83621815E-07
(3 1 0)	-0.25905794E-02	-0.16267199E-02	0.44903253E-03	0.24949716E-03	0.31811253E-04	0.12472213E-05
(3 1 1)	-0.93864940E-03	0.78226701E-03	0.65698746E-04	-0.76664572E-04	-0.13261908E-04	-0.58433454E-06
(3 1 2)	0.20464969E-03	-0.79798598E-04	-0.31340467E-04	0.26263592E-05	0.96852467E-06	0.50155474E-07
(3 2 0)	0.17937967E-03	-0.26284951E-03	-0.77522141E-04	0.32019542E-05	0.19907745E-05	0.11330622E-06
(3 2 1)	0.14379785E-03	0.11350385E-03	-0.52325278E-04	-0.22296259E-04	-0.25628334E-05	-0.93609758E-07
(3 2 2)	0.14439318E-04	0.13040934E-05	0.28224994E-05	0.82772388E-06	0.83029680E-07	0.27175665E-08
(3 3 0)	0.26685171E-04	0.27702134E-05	-0.91601637E-05	-0.29156552E-05	-0.30693882E-06	-0.10715202E-07
(3 3 1)	0.30442128E-04	0.55124161E-05	-0.77587641E-05	-0.21593223E-05	-0.19452234E-06	-0.57567222E-08
(3 3 2)	-0.13835817E-05	0.65511747E-06	0.75405027E-06	0.12656149E-06	0.65449343E-08	0.47775400E-10

Table 4: The coefficients $d_{i,j,k,l}^\alpha$ for perpendicular diffusion in the NLGCE-F model.

(j k l)	i=0	i=1	i=2	i=3	i=4	i=5
(0 0 0)	-0.20782397E+01	0.80542173E+00	0.43509413E-01	-0.21765985E-01	-0.34880893E-02	-0.14567092E-03
(0 0 1)	-0.46386191E+00	-0.98573919E-01	0.55008025E-02	0.60611018E-02	0.81158994E-03	0.32584425E-04
(0 0 2)	-0.20027545E-01	0.77767273E-02	-0.17837106E-03	-0.34531413E-03	-0.48546308E-04	-0.20284890E-05
(0 1 0)	0.32932663E-01	0.55078257E-01	0.57859672E-02	-0.98263229E-02	-0.15658018E-02	-0.64502352E-04
(0 1 1)	0.79801317E-01	-0.16056809E-01	-0.43059925E-02	0.15297052E-02	0.29837887E-03	0.13184386E-04
(0 1 2)	-0.96094000E-02	-0.70750559E-03	0.71313130E-04	-0.14638662E-03	-0.27273546E-04	-0.12478895E-05
(0 2 0)	0.45254433E-01	-0.15503612E-03	-0.58120004E-02	-0.95055489E-03	-0.41356187E-04	0.75481060E-07
(0 2 1)	-0.12156453E-01	-0.14601653E-02	0.16977702E-02	0.48775342E-03	0.45538505E-04	0.14225158E-05
(0 2 2)	0.14840409E-02	0.42407064E-03	-0.45749343E-04	-0.31128841E-04	-0.36845885E-05	-0.13458210E-06
(0 3 0)	0.32974684E-02	-0.60751030E-03	-0.81092420E-03	-0.65981454E-04	0.46104859E-05	0.42766278E-06
(0 3 1)	-0.10773557E-02	-0.29570722E-03	0.43219766E-04	0.13131432E-05	-0.13053938E-05	-0.84558084E-07
(0 3 2)	0.12302544E-03	0.56007217E-04	0.10139352E-04	0.24540202E-05	0.33361447E-06	0.14524518E-07
(1 0 0)	-0.11101067E+01	0.31228772E+00	-0.13177282E-01	-0.22255326E-01	-0.30794494E-02	-0.12445252E-03
(1 0 1)	-0.27352296E+00	-0.13632322E+00	0.95260785E-02	0.80307990E-02	0.96089118E-03	0.35198094E-04
(1 0 2)	0.12818726E-01	0.96450258E-02	-0.23937203E-02	-0.11687264E-02	-0.13273061E-03	-0.47474106E-05
(1 1 0)	-0.10207300E+00	0.11063794E-01	0.55260570E-02	-0.14631952E-02	-0.28667936E-03	-0.12195614E-04
(1 1 1)	-0.89940234E-02	0.15794357E-01	0.10450265E-01	0.23173513E-02	0.20829148E-03	0.66426911E-05
(1 1 2)	0.32828567E-02	-0.10346441E-02	-0.10776541E-02	-0.22747122E-03	-0.19402946E-04	-0.60184518E-06
(1 2 0)	0.16729966E-01	-0.86926828E-02	-0.47449408E-02	-0.25011753E-03	0.52349008E-04	0.39216750E-05
(1 2 1)	-0.68420124E-02	0.15559548E-02	-0.95408753E-03	-0.59427782E-03	-0.82159092E-04	-0.33850261E-05
(1 2 2)	0.80345403E-03	-0.54456722E-04	0.18658915E-03	0.84258499E-04	0.10778557E-04	0.42789250E-06
(1 3 0)	0.22208796E-02	-0.58431586E-03	-0.48924660E-03	-0.27928743E-04	0.51958515E-05	0.39174790E-06
(1 3 1)	-0.38128951E-04	0.13477913E-03	-0.25264265E-03	-0.10906523E-03	-0.13641539E-04	-0.53623767E-06
(1 3 2)	0.87033985E-05	-0.14763804E-04	0.25957223E-04	0.11060270E-04	0.13830492E-05	0.54549206E-07
(2 0 0)	-0.35795231E+00	0.65313508E-01	-0.50976726E-02	-0.52759476E-02	-0.70390662E-03	-0.28026723E-04
(2 0 1)	-0.80001226E-01	-0.34222505E-01	0.15934024E-02	0.15341728E-02	0.17475526E-03	0.60919963E-05
(2 0 2)	0.87639890E-02	0.34113860E-02	-0.18169113E-03	-0.16835979E-03	-0.18690148E-04	-0.62524682E-06
(2 1 0)	-0.26504203E-02	-0.19821713E-02	0.26199589E-02	0.52733957E-03	0.46965191E-04	0.16425008E-05
(2 1 1)	0.31124751E-03	0.71374069E-02	0.19284097E-02	0.75522804E-04	-0.14569881E-04	-0.96859723E-06
(2 1 2)	0.55387018E-03	-0.72916289E-03	-0.22397964E-03	-0.51781606E-06	0.30976122E-05	0.17280133E-06
(2 2 0)	0.25359898E-02	-0.29959443E-02	-0.89093469E-03	0.56975030E-04	0.25922842E-04	0.14212540E-05
(2 2 1)	-0.77368564E-03	0.92068559E-03	-0.29143432E-03	-0.18699531E-03	-0.25019943E-04	-0.10043456E-05
(2 2 2)	0.16753641E-03	-0.63376050E-04	0.32826799E-04	0.18838799E-04	0.24833739E-05	0.98856519E-07
(2 3 0)	0.37522544E-03	-0.17359581E-03	-0.84952992E-04	0.16008488E-05	0.19086792E-05	0.11081771E-06
(2 3 1)	0.77690104E-04	0.52930691E-04	-0.71571141E-04	-0.28796227E-04	-0.34496071E-05	-0.13183005E-06
(2 3 2)	-0.33219791E-05	-0.36269908E-05	0.64832962E-05	0.24768957E-05	0.29318259E-06	0.11173399E-07
(3 0 0)	-0.26370968E-01	0.45853683E-02	-0.42857969E-03	-0.38478482E-03	-0.50492454E-04	-0.19949740E-05
(3 0 1)	-0.59333101E-02	-0.24985017E-02	0.83613754E-04	0.93963534E-04	0.10360895E-04	0.34793936E-06
(3 0 2)	0.82864537E-03	0.29126396E-03	0.91218775E-05	-0.58268328E-05	-0.64542109E-06	-0.18840624E-07
(3 1 0)	-0.21245015E-02	-0.24321040E-03	0.27708187E-03	0.76882914E-04	0.81206123E-05	0.30009780E-06
(3 1 1)	0.21504225E-03	0.58749970E-03	0.68725126E-04	-0.23819124E-04	-0.45232048E-05	-0.20090275E-06
(3 1 2)	0.13663304E-04	-0.67148914E-04	-0.80040661E-05	0.38261029E-05	0.69827212E-06	0.30722829E-07
(3 2 0)	0.86363838E-04	-0.25412269E-03	-0.47306183E-04	0.10707095E-04	0.25867542E-05	0.12762594E-06
(3 2 1)	0.69686411E-05	0.95111050E-04	-0.24784177E-04	-0.15707547E-04	-0.20542771E-05	-0.81082411E-07
(3 2 2)	0.86898718E-05	-0.74726285E-05	0.15429839E-05	0.12006276E-05	0.16251525E-06	0.64782353E-08
(3 3 0)	0.18940167E-04	-0.14961780E-04	-0.47045372E-05	0.55656921E-06	0.17796779E-06	0.91749454E-08
(3 3 1)	0.10317043E-04	0.54645836E-05	-0.52425736E-05	-0.20926819E-05	-0.24595118E-06	-0.92493584E-08
(3 3 2)	-0.50524289E-06	-0.31899297E-06	0.39480448E-06	0.14383851E-06	0.16356627E-07	0.60444987E-09

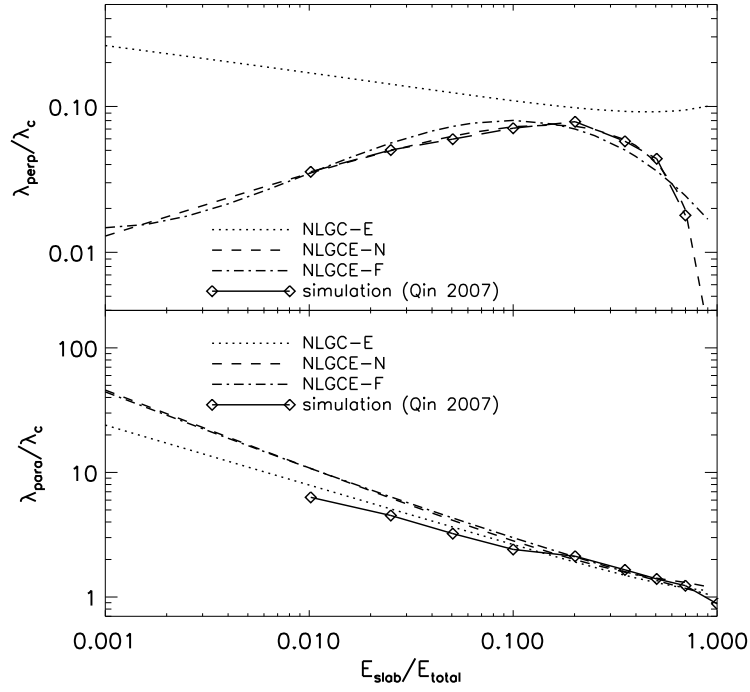


Fig. 1.— Top and bottom panels show perpendicular and parallel mean free paths, respectively, as a function of E_{slab}/E_{total} , with $r_L/\lambda_c = 0.048$, $b/B_0 = 1$ and $\lambda_{2D}/\lambda_{slab} = 0.1$. Diamonds are from the simulations in Qin (2007), Dotted, dashed, and dashed-dotted lines indicate results from NLGC-E, NLGCE-N, and NLGCE-F, respectively.

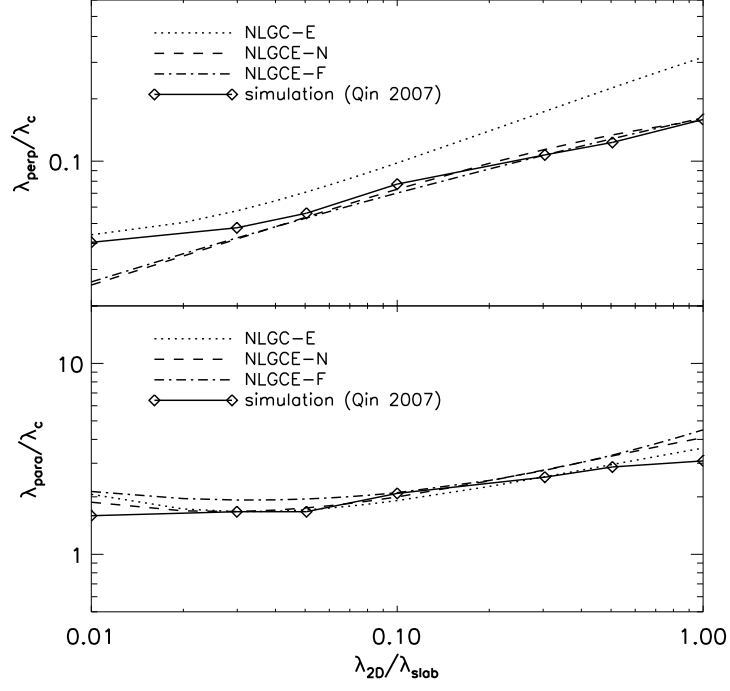


Fig. 2.— Similar as Figure 1, except that x-axis is $\lambda_{2D}/\lambda_{slab}$ with $E_{slab}/E_{total} = 0.2$.

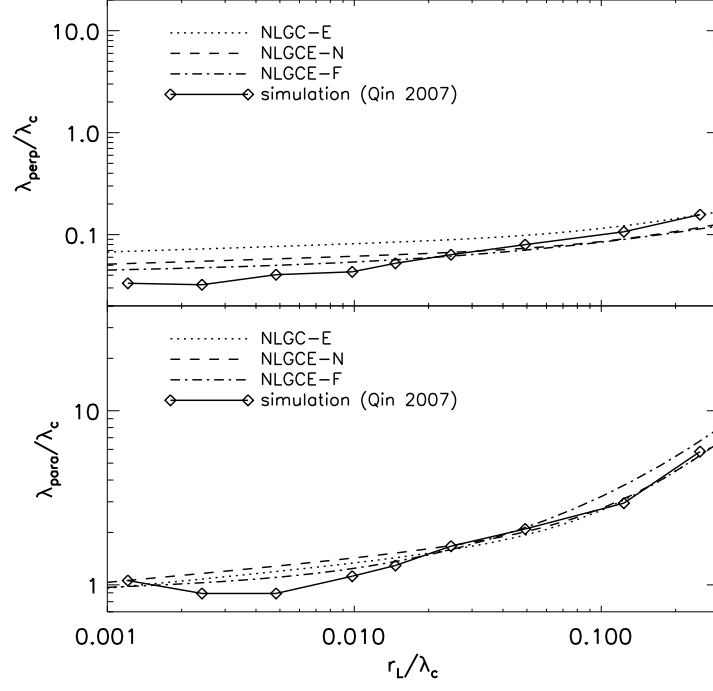


Fig. 3.— Similar as Figure 1, except that x-axis is r_L/λ_c with $E_{\text{slab}}/E_{\text{total}} = 0.2$.

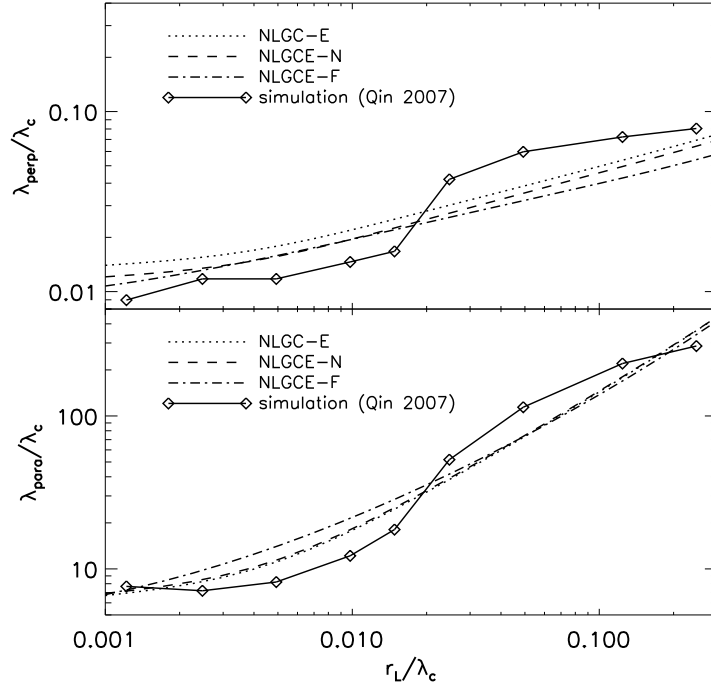


Fig. 4.— Similar as Figure 1, except that x-axis is r_L/λ_c with $E_{\text{slab}}/E_{\text{total}} = 0.2$ and $b/B_0 = 0.2$.

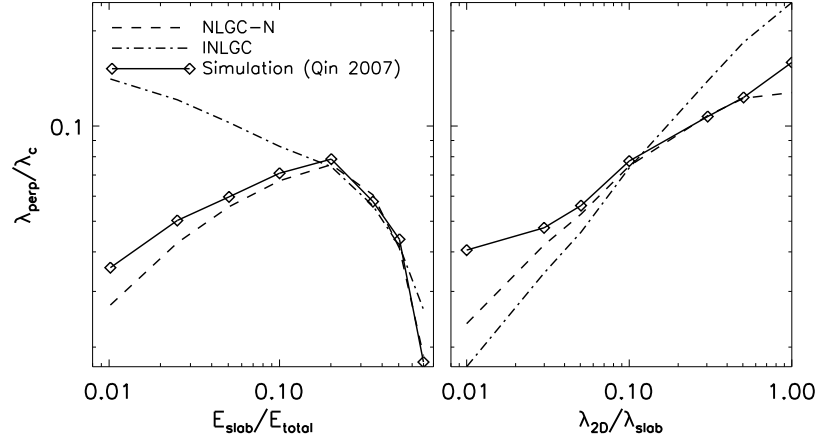


Fig. 5.— Left and right panels are similar as top panels of Figure 1 and Figure 2, respectively, except that dashed and dashed-dotted lines indicate results from NLGC-N and INLGC, respectively.

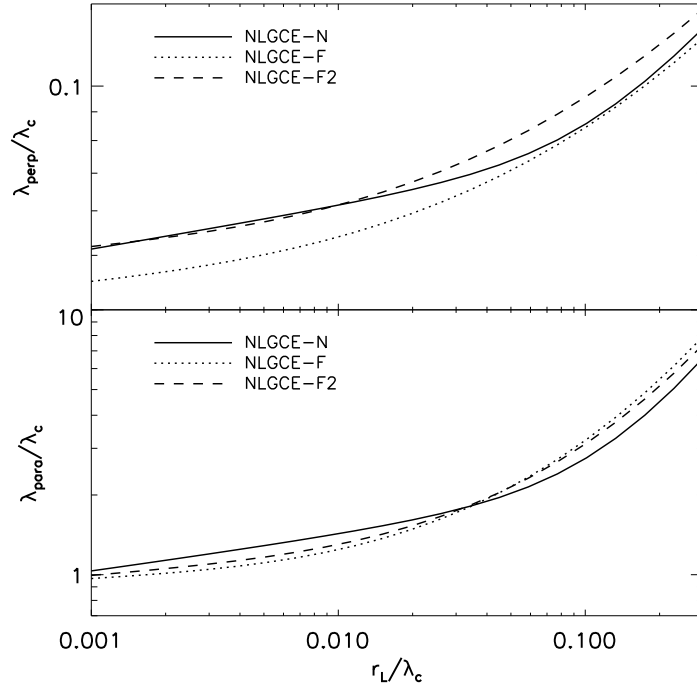


Fig. 6.— Similar as Figure 3, except that solid, dotted, and dashed lines indicate results from NLGCE-N, NLGCE-F, and NLGCE-F2, respectively.